**CME – Assignment 2**

**Exercise I**

Suppose we want to test:

If the unrestricted model is,

Then the restricted model is going to be,

Thus, we must drop the second column of the matrix X to derive OLS estimator for that satisfies to null.

**Part A**

**Part B**

**Part C**

**Part D**

**Part E**

|  |  |
| --- | --- |
| Theoretical Size | 17.02% |
| Residual Nonparametric Bootstrap Size | 7.68% |
| Pairs Bootstrap Size | 7.32% |
| Wild Bootstrap with Random Normal Size | 8.89% |
| Wild Bootstrap with Two Point Size | 5.08% |

**Exercise 2**

Rewritten AR(2) model for the Dickey-Fuller test,

to estimate and .

**Part A**

**Part B**

**Part C**

**Part D**

**Part E**

|  |  |  |
| --- | --- | --- |
| **Method/Data** | Timeseries\_het.txt | Timeseries\_dep.txt |
| Theoretical Size | 7.2% | 73.8% |
| Residual Nonparametric Bootstrap Size | 6.8% | 73.5% |
| Wild Bootstrap with Random Normal Size | 5.6% | 71.8% |
| Wild Bootstrap with Two Point Size | 4.8% | 69.7% |
| Block Bootstrap Size | 2.8% | 4.5% |
| Sieve Bootstrap Size | 1.9% | 3.6% |

**Literature about Bootstrap Methods**

The residual bootstrap

If the error terms in (8) are independent and identically distributed with common variance σ 2 , then we can generally make very accurate inferences by using the residual bootstrap. We do not need to assume that the errors follow the normal distribution or any other known distribution.

The wild bootstrap

The residual bootstrap is not valid if the error terms are not independently and identically distributed, but two other commonly used bootstrap methods are valid in this case. The first of these is the “wild bootstrap,” which was proposed by Wu (1986) for regression models with heteroskedastic errors. There are various ways to specify the distribution of the . (Two point or Random Normal)

The pairs bootstrap

Another method that can accommodate heteroskedasticity is the “pairs bootstrap.The idea is to resample the data instead of the residuals. Unlike the residual and wild bootstraps, the pairs bootstrap does not condition on X. Instead, each bootstrap sample has a different matrix.

The sieve bootstrap

Suppose that the error terms ut in a regression model, which for simplicity we may assume to be the linear regression model, follow an unknown, stationary process with homoskedastic innovations. The sieve bootstrap attempts to approximate this process, generally by using an AR(p) process with p chosen either by some sort of model selection criterion or by sequential testing.

Block bootstrap methods

Block bootstrap methods divide the quantities that are being resampled, which might be either rescaled residuals or [y, X] pairs, into blocks of b consecutive observations. The blocks, which may be either overlapping or nonoverlapping and may be either fixed or variable in length, are then resampled. It appears that the best approach is to use overlapping blocks of fixed length. This is called the “moving-block bootstrap.”

However, the wild bootstrap performs about as well as it did when the test statistic was (21), and the pairs bootstrap also performs reasonably well for large sample sizes. In this case, the ability of the wild and pairs bootstraps to mimic the heteroskedasticity in the data is evidently critical.

Bootstrap DGPs for Dependent Data

All of the bootstrap DGPs that have been discussed so far treat the error terms (or the data, in the case of the pairs bootstrap) as independent. When that is not the case, these methods are not appropriate. In particular, resampling (whether of residuals or data) breaks up whatever dependence there may be and is therefore unsuitable for use when there is dependence. Numerous bootstrap DGPs for dependent data have been proposed. The two most popular approaches are the “sieve bootstrap” and the “block bootstrap.” The former attempts to model the dependence using a parametric model. The latter resamples blocks of consecutive observations instead of individual observations.

**Lecture Notes on Bootstrap Methods**

Residual Bootstrap

Residual Bootstrap as we use the residuals to setup the bootstrap EDF.

Bootstrapping heteroskedastic data

The bootstrap procedures described above all depend on the assumption that the underlying true data generating process is IID. However, very often we have to deal with data that does not satisfy this assumption. One of the more common problems you might encounter in regression models is that your error terms are not homoscedastic.

Pairs Bootstrap

We cannot use the residual bootstrap as the innovations are not identically distributed anymore. One method called the pairs bootstrap assumes that the joint distribution (yt; xt) is iid. We can then use the EDF of that distribution and draw random sample pairs.

Wild Bootstrap

A very different technique of simulating under heteroskedasticity is called the wild

bootstrap. Similar as in the residual bootstrap we estimate beta under the null to get a

restricted optimizer. The idea is then to simulate epsilon\_star with epsilon\_hat \* v\_star where V(t)IID(0,1).

3.4 Bootstrapping dependent data

The bootstrap DGPs that we have discussed so far are not valid when applied to models with dependent errors having an unknown pattern of dependence. For such models, we wish to specify a bootstrap DGP which generates correlated error terms that exhibit approximately the same pattern of dependence as the real errors, even though we do not know the process that actually generated the errors. There are two main approaches.

Sieve Bootstrap

The first approach is called the sieve bootstrap and is based on World's theorem. Autoregressive model for epsilon to derive eta and simulate it with EDF to re-estimate epsilon\*star and y\*star respectively.

Block Bootstrap

The second approach that we discuss to deal with dependent data is called the block bootstrap. The idea is to divide the quantities that are being resampled, which might be either rescaled residuals or [yt; xt] pairs, into blocks of ` consecutive observations, and then resample the blocks. There are many versions of the block bootstrap, the blocks may be either overlapping or nonoverlapping. The method that often works the best is with overlapping blocks and called the moving-blocks bootstrap.